

## Exponential and Logarithmic Functions

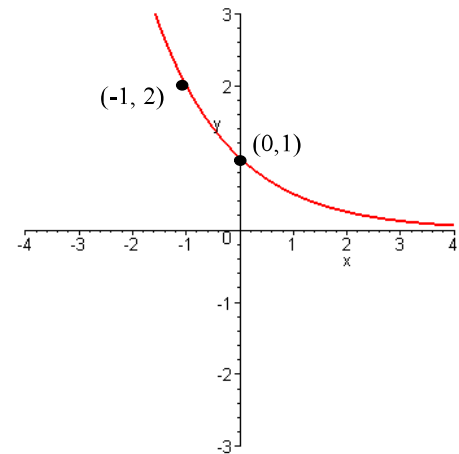
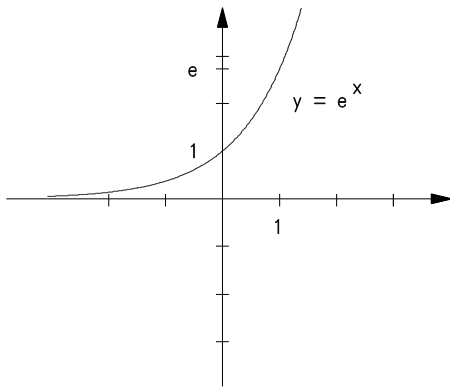
### Study Guide

Abadi, College Algebra

1. Know the definition of exponential functions:

$f(x) = b^x, b \neq 1, b > 0$  Here  $b$  is called the base of the exponential function.

The exponential functions  $y = e^x$  and  $y = \left(\frac{1}{2}\right)^x = 2^{-x}$  are graphed below. Here the base is  $e$ . Note that  $\left(1 + \frac{1}{x}\right)^x \rightarrow e$  when  $x \rightarrow \infty$ . (The value of  $e \approx 2.7183$ )



Exercise: Graph the following:

$$y = \left(\frac{1}{e}\right)^x = e^{-x}, \quad y = 3^x, \quad y = \left(\frac{1}{4}\right)^x.$$

Consider the exponential function  $f(x) = b^x, b \neq 1, b > 0$

- The domain of the exponential function is  $(-\infty, \infty)$
- The range of the exponential function is  $(0, \infty)$
- The y-intercept of the exponential is  $(0, 1)$
- The line  $y = 0$  is the horizontal asymptote.
- The graph is increasing if  $b > 1$  and decreasing if  $0 < b < 1$ .

2. Know how to use the following formulas:

$$i) A = P\left(1 + \frac{r}{k}\right)^{kt} \quad ii) A = Pe^{rt} \quad iii) A = A_0 2^{\frac{-t}{h}} \quad iv) P = P_0 2^{\frac{t}{d}}$$

**Logarithms:**

3.  $\log_b A = B$  is equivalent to  $A = b^B$ .

Here  $A > 0$ ,  $b > 0$ , and  $b \neq 1$ .

"In words,  $\log_b A$  is the exponent to which  $b$  must be raised in order to obtain  $A$ ."

Example:  $\log_2 8 = 3$  since  $2^3 = 8$

4. Laws Of Logarithms:

i)  $\log_b AB = \log_b A + \log_b B$

ii)  $\log_b \frac{A}{B} = \log_b A - \log_b B$

iii)  $\log_b A^C = C \cdot \log_b A$

iv)  $\log_b b = 1$

v)  $\log_b 1 = 0$

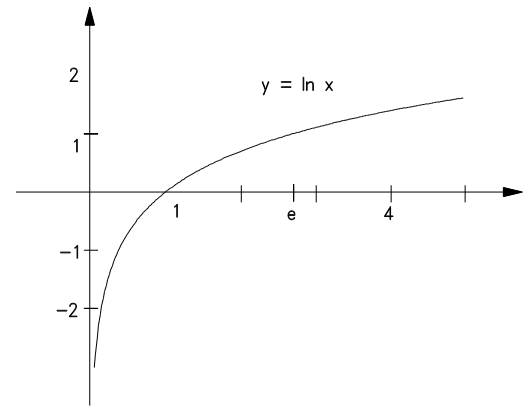
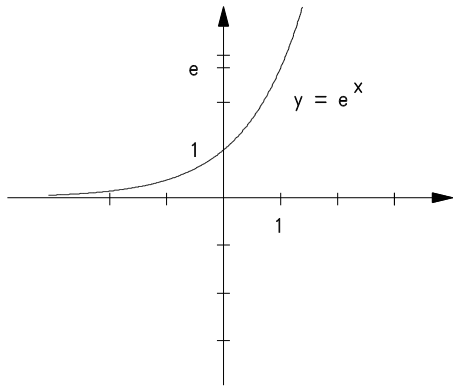
5. Logarithms with base 10 are called **common logarithms**.

$$\log x = \log_{10} x \quad \text{for every } x > 0$$

6. Logarithms with base  $e$  are called **natural logarithms**.

$$\ln x = \log_e x \quad \text{for every } x > 0$$

7. Change of Base Formula.  $\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}$



The exponential and Logarithmic functions are inverses of each other.

8. Consider the logarithmic function  $f(x) = \log_b x, b \neq 1, b > 0$

i) The domain of the logarithmic function is  $(0, \infty)$

ii) The range of the logarithmic function is  $(-\infty, \infty)$

iii) The  $x$ -intercept of the logarithmic function is  $(1, 0)$

iv) The line  $x = 0$  is the vertical asymptote.

v) The graph is increasing if  $b > 1$  and decreasing if  $0 < b < 1$ .

vi) Exponential Growth:  $P = P_0 e^{rt}, r > 0$       Exponential Decay:  $P = P_0 e^{rt}, r < 0$

