3-6 Variation and Modeling

Modeling with Direct Variation

Definition 1: Let x and y be variables. The statement **y is directly proportional to x** (or **y varies directly as x**) means

$$y = kx$$

for some nonzero constant k, called the **constant of proportionality** (or **constant of variation**).

Example 1: The force F exerted by a spring is directly proportional to the distance x that is stretched. (This is known as Hooke's law.) Find the constant of proportionality and the equation of variation if $F = 12$ pounds when $x = \frac{1}{3}$ foot.

Example 2: The distance d covered by a falling object is directly proportional to the square of the length of time t since it began falling. If a coin dropped from a suspension bridge falls 144 feet is 3 seconds, find the equation of variation, and use it to find how far the coin would fall in 5 seconds.

Modeling with Inverse Variation

Definition 2: Let x and y be variables. The statement **y is inversely proportional to x** (or **y varies inversely as x**) means

$$y = \frac{k}{x}$$

for some nonzero constant k, called the **constant of proportionality** (or **constant of variation**).

Example 3: The note played by each pipe in a pipe organ is determined by the frequency of vibration of air in the pipe. The fundamental frequency f of vibration of air in an organ pipe is inversely proportional to the length L of the pipe. (This is why the low-frequency notes come from the very long pipes.)

(A) Find the constant of proportionality and the equation of variation if the fundamental frequency of an 8-foot pipe is 64 variations per second.

(B) Find the fundamental frequency of a 1.6-foot pipe.

Modeling with Joint and Combined Variation

Let x, y, and w be variables. The statement ***w* is jointly proportional to x and y** (or ***w* varies jointly as x and y**) means

$$w = kxy$$

for some nonzero constant k, called the **constant of proportionality** (or **constant of variation**).

Example 4: The volume V of a right circular cone is jointly proportional to the square of its radius r and its height h. Find the constant of proportionality and the equation of variation if a cone of height 8 inches and radius 3 inches has a volume of 24π cubic inches.

Example 5: The note played by a string on a guitar is determined by the frequency at which the string vibrates. The frequency f of a vibrating string is directly proportional to the square root of the tension T and inversely proportional to the length L.

(A) If the tension of a guitar string is increased, how does the frequency change? What if the length is increased?

(B) What is the effect on the frequency if the length is doubled and the tension is quadrupled?