

7-6 The Binomial Formula

Using Pascal's Triangle

Consider $(a + b)^n$

$$(a + b)^0 = 1$$

$$(a + b)^1 = a + b$$

$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$

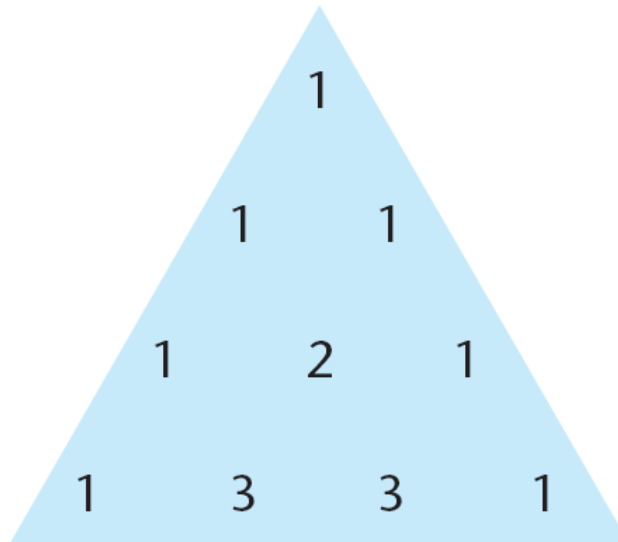
$$(a + b)^3 = (a + b)(a + b)(a + b) = a^3 + 3a^2b + 3ab^2 + b^3$$

*Notice: How many terms are in each expansion?

What is the first term of each?

What is the last term of each?

How are the coefficients related? Check out Pascal's triangle:



▶ **Figure 1** Pascal's triangle.

Example 1: Use Pascal's triangle to expand $(x + y)^4$.

The Binomial Formula

Definition 1: Combination Formula: For nonnegative integers r and n , $0 \leq r \leq n$,

$$\binom{n}{r} = C_{n,r} = \frac{n!}{r!(n-r)!}$$

Theorem 1: Binomial Formula: For n a positive integer

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

Example 2: Use the binomial formula to expand $(x + y)^6$.

Example 3: Use the binomial formula to expand $(3p - 2q)^4$.

Example 4: Find the term containing x^9 in the expansion of $(x + 3)^{14}$.

Example 5: If the terms in the expansion of $(x - 2)^{20}$ are arranged in decreasing powers of x , find the fourth term and the sixteenth term.

*Note: In the expansion $(a + b)^n$, the exponent of b in the r th term is $r - 1$ and the exponent of a is $n - (r - 1)$.

Example of Homework #63: Evaluate $(1.01)^{10}$ to four decimal places, using the binomial formula.