Differential Equations & Multi-Dimensional Calculus, Math 3525
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Maximum & Minimum Values

1. $z = f(x,y)$

i) $f(a, b)$ is a local maximum or local minimum

ii) $f_x(a,b)$ & $f_y(a,b)$ exist

$$\Rightarrow f_x(a,b) = 0 \;\&\; f_y(a,b) = 0$$

2. A point (a, b) is called a **critical point** if $f_x(a,b) = 0$ & $f_y(a,b) = 0$ or one these partial derivatives does not exit.

3. $z = f(x,y)$

i) $f_{xx}(x,y), f_{yy}(x,y)$ & $f_{xy}(x,y)$ are continuous on a disk centered at (a, b),

ii) $f_x(a,b) = 0$ & $f_y(a,b) = 0$; *i.e* (a, b) is a critical point of f,

iii) $D = D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$

To remember D, use the following determinant $D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx} f_{yy} - \left(f_{xy}\right)^2$

a) If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a, b)$ is a local minimum.

b) If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a, b)$ is a local maximum.

c) If $D < 0$, then $f(a, b)$ is not a local minimum or maximum.

Note 1: In case (c) the point (a, b) is called a **saddle point** of f and the tangent plane at (a, b) crosses the graph of f

Note 2: If $D = 0$, the test gives no information; *i.e* f could have a local minimum or local maximum at (a, b), or (a, b) could be a saddle point of f.