

Differential Equations with Multi-Dimensional Calculus, Math 3525

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Basic Integration Formulas:

$$1. \quad \int x^r dx = \begin{cases} \frac{1}{r+1} x^{r+1} & \text{if } r \neq -1 \\ \ln x & \text{if } r = -1 \end{cases} \quad (\text{Power Rule})$$

$$2. \quad \int \sin x dx = -\cos x + C \qquad \int \cos x dx = \sin x + C$$

$$3. \quad \int \tan x dx = \int \frac{\sin x}{\cos x} dx = -\ln|\cos x| + C = \ln|\sec x| + C$$

$$4. \quad \int \sec^2 x dx = \tan x + C \qquad \int \sec x \tan x dx = \sec x + C$$

$$5. \quad \int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1 \quad (\text{Power Rule})$$

$$6. \quad \int \frac{du}{u} = \ln|u| + C$$

$$7. \quad \int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C = \sin^{-1} x + C$$

$$8. \quad \int \frac{1}{1+x^2} dx = \arctan x + C = \tan^{-1} x + C$$

$$9. \quad \int e^u du = e^u + C$$

$$10. \quad \int u dv = uv - \int v du \quad (\text{Integration by Parts})$$

Example:

$$\int x \ln x dx: \begin{cases} u = \ln x & dv = x \\ du = \frac{1}{x} & v = \frac{1}{2} x^2 dx \end{cases}$$

$$\int x \ln x dx = uv - \int v du = \frac{1}{2} x^2 \ln x - \int x \cdot \frac{1}{x} dx$$

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$$11. \quad \int \frac{1}{(x+1)(x-2)} dx = \int \frac{-\frac{1}{3}}{x+1} dx + \int \frac{\frac{1}{3}}{x-2} dx = -\frac{1}{3} \ln|x+1| + \frac{1}{3} \ln|x-2| + C \quad (\text{Partial Fractions})$$