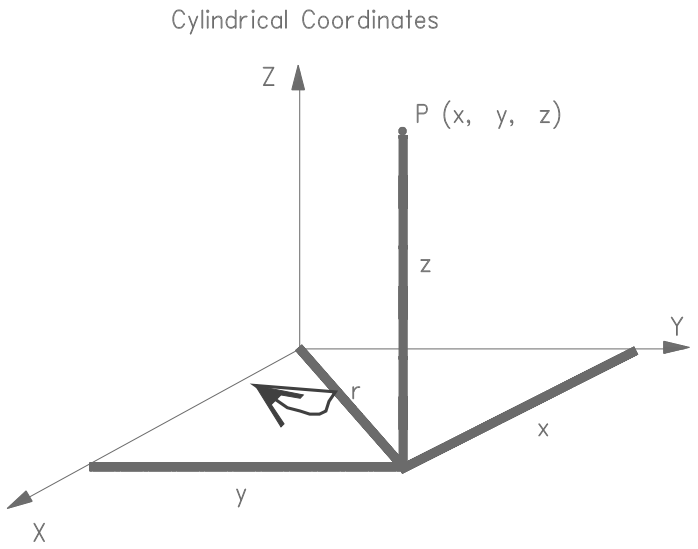
Math 3543, Calculus III Name.....

$$P(r, \theta, z) = P(x, y, z) = P(\rho, \theta, \phi)$$

Relationship between Cylindrical & Cartesian coordinates

a) $$\begin{cases} r^2 = x^2 + y^2 \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) \\ z = z \end{cases}$$

b) $$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$



Restriction: $0 \le \theta < 2\pi$

$r = 2$ is the vertical cylinder

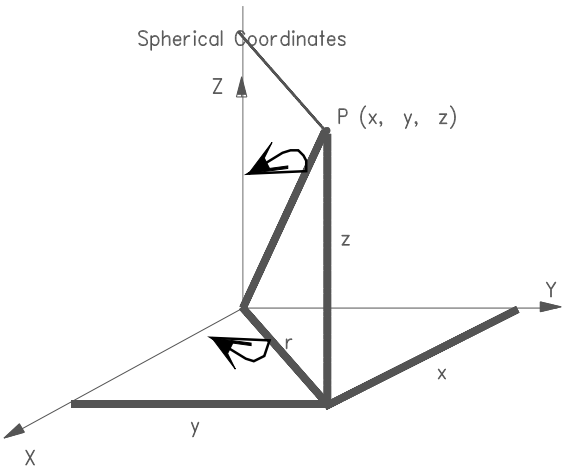
$x^2 + y^2 = 4$.

Relationship between Cylindrical & Spherical coordinates and Spherical & Cartesian coordinates.

Restrictions: $\rho \ge 0$, $0 \le \theta < 2\pi$, $0 \le \phi \le \pi$

a) $r = \rho\sin\phi$, $\theta = \theta$, $z = \rho\cos\phi$,

b) $$\begin{cases} \rho = \sqrt{x^2 + y^2 + z^2} \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) \\ \phi = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right) \end{cases}$$

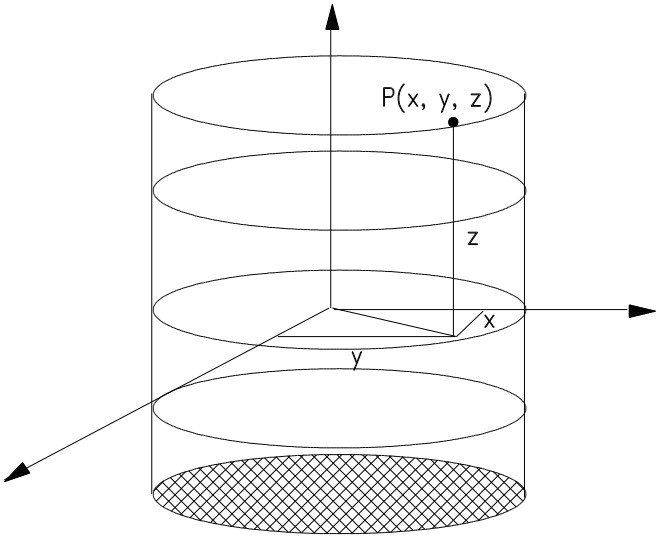


c) $\begin{cases} x = \rho \sin\phi\cos\theta \\ y = \rho \sin\phi\sin\theta \\ z = \rho\cos\phi \end{cases}$

d) $\rho = \sqrt{r^2+z^2}$ $\quad \theta = \theta \quad$ $\phi = \cos^{-1}\dfrac{z}{\sqrt{r^2+z^2}}$

$\rho = 1$ is the unit sphere $\quad x^2+y^2+z^2=1.$

Examples:



i) $r = 2$ is the vertical cylinder

$x^2+y^2=4.$

ii) $r = \sqrt{4-z^2}$ is the unit sphere

$x^2+y^2+z^2=1.$

iii) $\rho = 2\csc\phi$ is the vertical cylinder

$x^2+y^2=4.$

iv) $\rho = 1$ is the unit sphere

$x^2+y^2+z^2=1.$