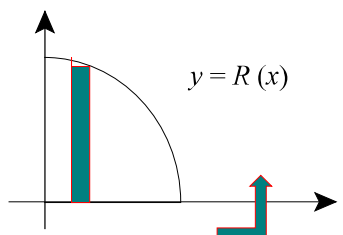
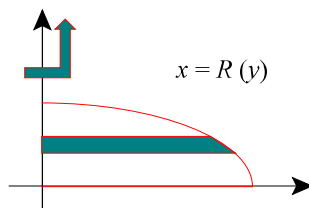


1. Volume: $V = \pi \int_a^b [R(x)]^2 dx$

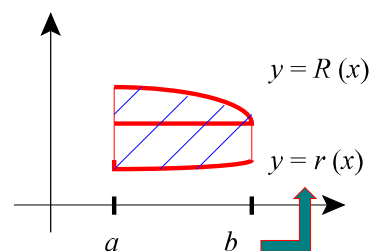


$V = \pi \int_c^d [R(y)]^2 dy$

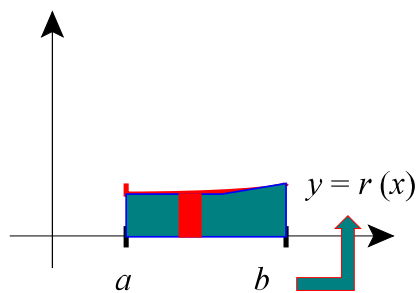
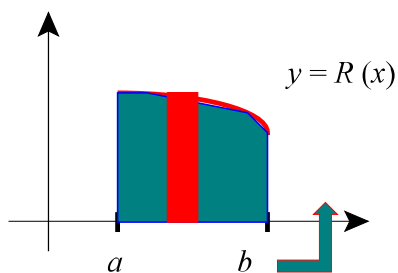
Disk Method



2. $V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2) dx$



Washer Method



$V_1 = \pi \int_a^b [R(x)]^2 dx$

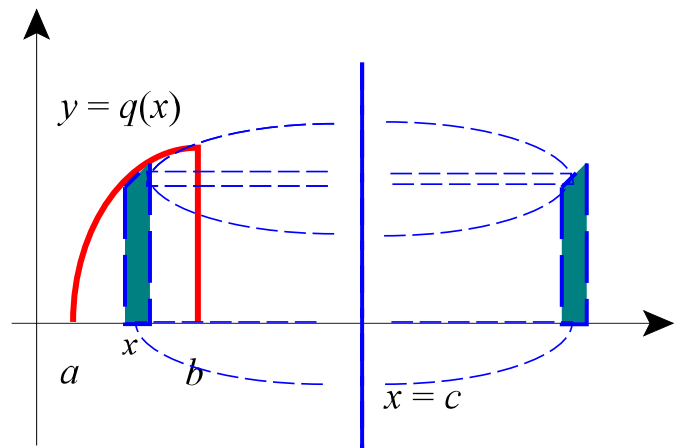
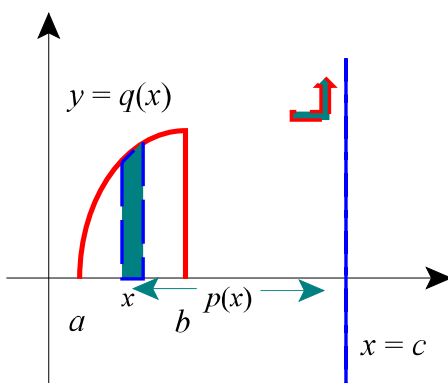
$V_2 = \pi \int_a^b [r(x)]^2 dx$

3.
$$V = 2\pi \int_c^d p(y)q(y)dy$$

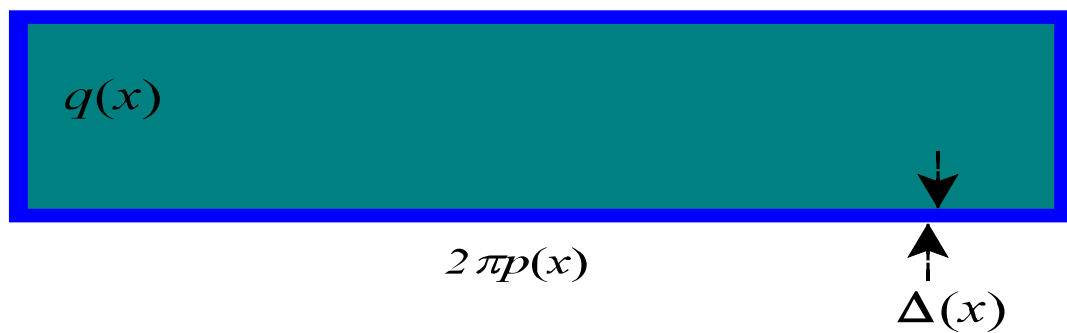
$$V = 2\pi \int_a^b p(x)q(x)dx$$

Shell Method

Note that here $p(x) = c - x$



“Cut the above shell open on one side to get the following rectangular sheet with thickness Δx , height $q(x)$, and length $2\pi \cdot p(x)$ (the circumference of the circle)”



$$4. \quad \text{Arc Length:} \quad s = \int_a^b \sqrt{1+[f'(x)]^2} dx \qquad s = \int_c^d \sqrt{1+[g'(y)]^2} dy$$

$$5. \quad \text{Arc Length:} \quad \begin{cases} x = f(t) \\ y = g(t) \end{cases} a \leq t \leq b; \quad s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$6. \quad \text{Surface Area:} \quad s = 2\pi \int_a^b r(x) \sqrt{1+[f'(x)]^2} dx \qquad s = 2\pi \int_c^d r(y) \sqrt{1+[g'(y)]^2} dy$$

Some Useful Formulas:

$$1. \quad \int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + c$$

$$2. \quad \int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + c$$

$$3. \quad \int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + c$$

$$4. \quad \int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln(u + \sqrt{u^2 \pm a^2}) + c$$

$$5. \quad \int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{a+u}{a-u} \right| + c$$

$$6. \quad \int \frac{du}{u\sqrt{a^2 \pm u^2}} = -\frac{1}{a} \ln \frac{a + \sqrt{a^2 \pm u^2}}{|u|} + c$$