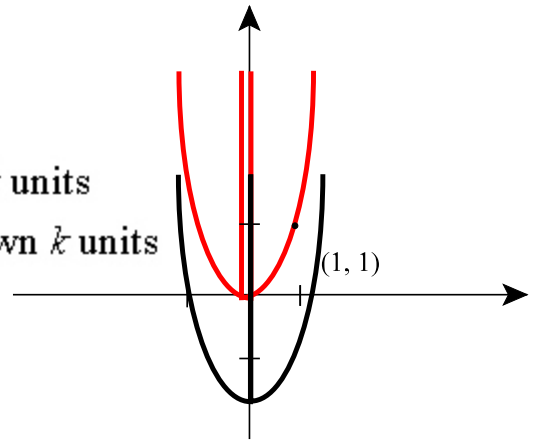


Calculus I, Graph Transformations
Abedi

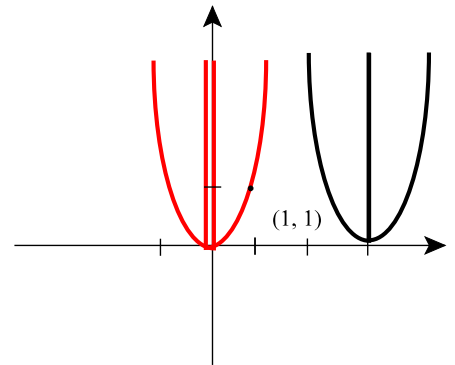
1. Vertical Translation:

$$y = f(x) + k \begin{cases} k > 0 & \text{Shift the graph of } y = f(x) \text{ up } k \text{ units} \\ k < 0 & \text{Shift the graph of } y = f(x) \text{ down } k \text{ units} \end{cases}$$



2. Horizontal Translation:

$$y = f(x - k) \begin{cases} k > 0 & \text{Shift the graph of } y = f(x) \text{ right } k \text{ units} \\ k < 0 & \text{Shift the graph of } y = f(x) \text{ left } k \text{ units} \end{cases}$$



3. Vertical Expansion:

$$y = af(x) \begin{cases} a > 1 & \text{Vertically expand(stretch) the graph of } y = f(x) \text{ by multiplying each } y \text{ value by } a \\ 0 < a < 1 & \text{Vertically contract(compress) the graph of } y = f(x) \text{ by multiplying each } y \text{ value by } a \end{cases}$$

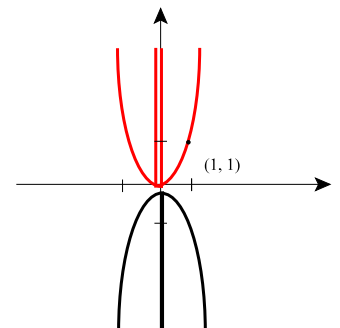
4. *Horizontal Expansion:

$$y = f(bx) \begin{cases} b > 1 & \text{Horizontally contract(compress) the graph of } y = f(x) \text{ by multiplying each } x \text{ value} \\ 0 < b < 1 & \text{Horizontally expand(stretch) the graph of } y = f(x) \text{ by multiplying each } x \text{ value} \end{cases}$$

5. Reflection:

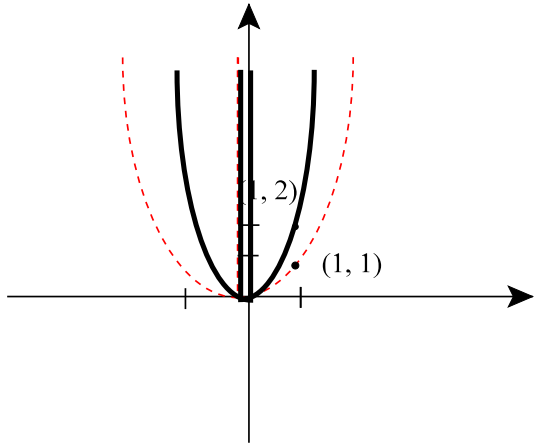
$$y = -f(x) \quad \text{Reflect the graph of } y = f(x) \text{ across the } x\text{-axis}$$

$$* y = f(-x) \quad \text{Reflect the graph of } y = f(x) \text{ across the } y\text{-axis}$$



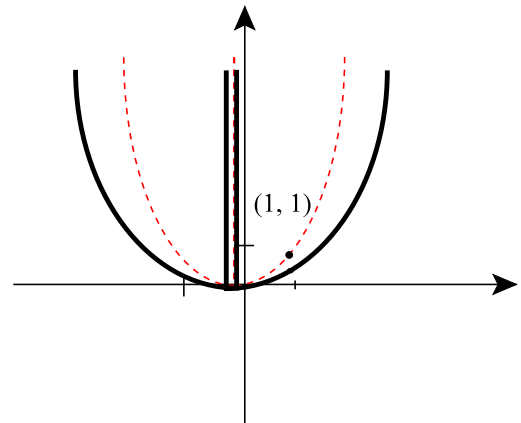
Example 1: To graph $y = 2x^2$, double the units along the y -axis.

(Expand $y = x^2$ vertically)



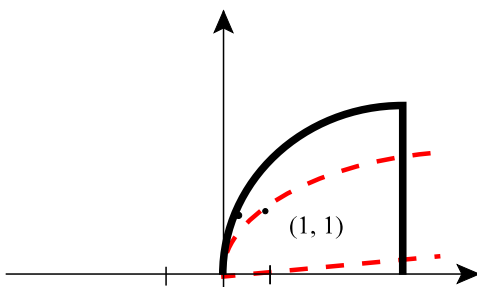
Example 2: To graph $y = \frac{1}{2}x^2$, half the units along the y -axis.

(Contract $y = x^2$ vertically)



Example 3: To graph $y = \sqrt{2x}$, half the units along the x -axis.

(Compress $y = \sqrt{x}$ Horizontally)



$$\sqrt{x} = y$$

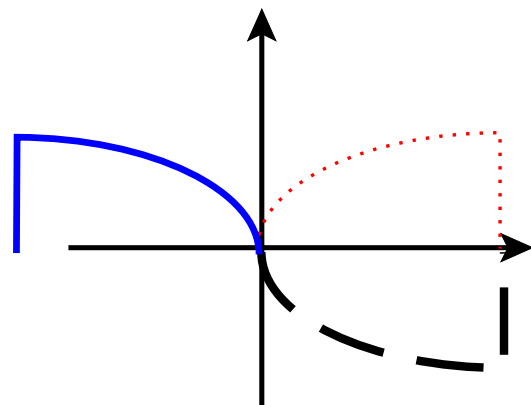
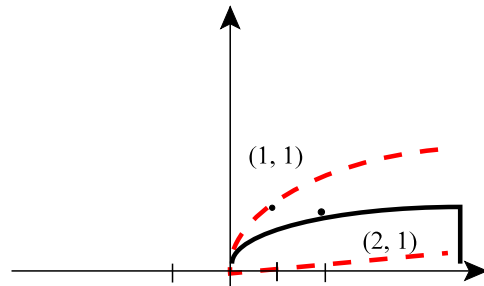
$$\sqrt{x/2} = y$$

Example 5: To graph $y = -\sqrt{x}$, reflect the graph of $y = \sqrt{x}$ in the x -axis.

To graph $y = \sqrt{-x}$, reflect the graph of $y = \sqrt{x}$ in the y -axis.

Example 4: To graph $y = \sqrt{\frac{1}{2}x}$, double the units along the x -axis.

(Expand $y = \sqrt{x}$ Horizontally)

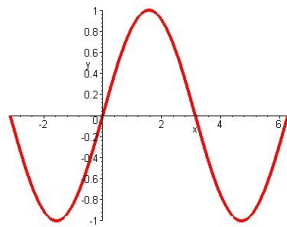


Example 6: To graph $y = \sin 2x$, half the units along the x -axis as compare to the graph of $y = \sin x$.

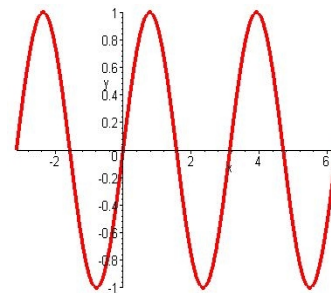
(Compress $y = \sin x$ Horizontally). Note that the period of $y = \sin x$ is 2π , while the period of $y = \sin 2x$ is π .

To graph $y = \sin(\frac{1}{2}x)$, double the units along the x -axis as compare to the graph of $y = \sin x$.

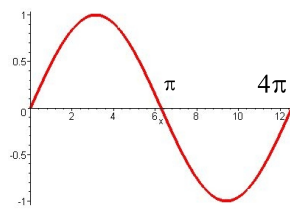
(Stretch $y = \sin x$ Horizontally). Note that the period of $y = \sin x$ is 2π , while the period $y = \sin(\frac{1}{2}x)$ is 4π .



$$y = \sin x$$



$$y = \sin 2x$$



$$y = \sin\left(\frac{1}{2}x\right)$$