Calculus II, Math 3495 Name...
Abedi

i) Power Series $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots\cdots + x^n + \cdots\cdots$ converges to $\dfrac{1}{1-x}$ if $|x| < 1$.

Radius of Convergence: $R = 1$ Interval of Convergence : $(-1, 1)$

ii) The power series $\sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + \ldots\ldots + (-1)^n x^n + \ldots$ converges to $\dfrac{1}{1+x}$ if $|x| < 1$,

and $\int \frac{1}{1+x}\,dx = \ln(x+1) + c$, $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots \frac{(-1)^n x^n}{n} \cdots = \sum_{n=1}^{\infty} \dfrac{(-1)^{n+1} x^n}{n}$, $-1 < x \le 1$

(Note that you have to check the convergency at the endpoints) Interval of Convergence : $(-1, 1]$

Note that : $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots - \frac{x^n}{n} \cdots = -\sum_{n=1}^{\infty} \frac{x^n}{n}$, $-1 \le x < 1$

iii) Power Series $\sum_{n=0}^{\infty} (-1)^n x^{2n} = 1 - x^2 + x^4 - x^6 + \cdots\cdots + (-1)^n x^{2n} + \cdots\cdots$ converges to $\dfrac{1}{1+x^2}$ if $|x| < 1$.

Radius of Convergence: $R = 1$ Interval of Convergence : $(-1, 1)$

iv) Since $\dfrac{d}{dx}(\arctan x) = \dfrac{1}{1+x^2}$ the power series

$$\int \frac{1}{1+x^2}\,dx = \sum_{n=0}^{\infty} \tfrac{(-1)^n}{2n+1} x^{2n+1} = x - \tfrac{1}{3}x^3 + \tfrac{1}{5}x^5 - \tfrac{1}{7}x^7 + \cdots\cdots + \tfrac{(-1)^n}{2n+1} x^{2n+1} + \cdots\cdots$$

converges to $\arctan x = \tan^{-1} x$ if $|x| < 1$. This fallows from $\arctan x = \int \frac{1}{1+x^2}\,dx$ and (iii) above.

(Note that you have to check the convergency at the endpoints) Interval of Convergence : $[-1, 1]$

v)

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \tfrac{1}{2}x^2 + \tfrac{1}{6}x^3 + \tfrac{1}{4!}x^4 + \cdots\cdots + \tfrac{1}{n!}x^n + \cdots\cdots, -\infty < x < \infty$$

vi)

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \tfrac{1}{3!}x^3 + \tfrac{1}{5!}x^5 - \tfrac{1}{7!}x^7 + \cdots\cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots\cdots, -\infty < x < \infty$$

vii)

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \tfrac{1}{2!}x^2 + \tfrac{1}{4!}x^4 - \tfrac{1}{6!}x^6 + \cdots\cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots\cdots, -\infty < x < \infty$$