

1. Find the radius of convergence and interval of convergence of the following power series

i) $\sum_{n=1}^{\infty} 5^n x^n$ ii) $\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{n+1}$ iii) $\sum_{n=1}^{\infty} \frac{x^n}{n^n}$ iv) $\sum_{n=2}^{\infty} 3 \frac{4^n x^n}{5^n}$

2. Find a power series representation for the function $f(x) = \frac{x^2}{4+x}$ and $f(x) = \frac{1}{2-x^2}$.

3. Evaluate the indefinite integral $\int \frac{1}{1-x^4} dx$ as a power series. Approximate $\int_0^1 \sqrt{1-x^4} dx$.

4. i) Find the radius of convergence and interval of convergence of the power series

$$f(x) = \sum_{n=1}^{\infty} \frac{(-2)^n x^n}{n!}.$$

- ii) Use Taylor's Inequality to estimate the accuracy of the approximation $f(x)$ using $T_3(x)$ where $x = 1$. (Hint; ask me about it, if needed)

5. Find the Maclaurin series for f and its radius of convergence.

i) $f(x) = \frac{x^3}{1-x^2}$ ii) $f(x) = \tan^{-1}(2x)$ iii) $f(x) = \ln(1-x^2)$

iv) $f(x) = x^3 e^{3x}$ $f(x) = (1-2x)^{-4}$ vi) $f(x) = \frac{x}{\cos x}$

6. a) Approximate f by a Taylor polynomial with degree n centered at $x = a$.
 b) Use Taylor's Inequality to estimate the accuracy of the approximation

$f(x) \approx T_n(x)$ where x lies in the given interval.

- c) Check your result in part (b) by graphing $|R_n(x)|$. (Hint; $|R_n(x)| = |f(x) - T_n(x)|$)

i) $f(x) = \sqrt{x}$ $a = 2$ $n = 2$ $-19 \leq x \leq 21$

ii) $f(x) = x \cos x$ $a = 0$ $n = 4$ $-1 \leq x \leq 1$

iii) $f(x) = x \ln x$ $a = 1$ $n = 4$ $1 \leq x \leq 15$

iv) $f(x) = x e^{-x}$ $a = -1$ $n = 3$ $-1 \leq x \leq 0$