

Graph Transformations

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1. Vertical Translation:

$$y = f(x) + k \begin{cases} k > 0 & \text{Shift the graph of } y = f(x) \text{ up } k \text{ units} \\ k < 0 & \text{Shift the graph of } y = f(x) \text{ down } k \text{ units} \end{cases}$$

2. Horizontal Translation:

$$y = f(x - k) \begin{cases} k > 0 & \text{Shift the graph of } y = f(x) \text{ right } k \text{ units} \\ k < 0 & \text{Shift the graph of } y = f(x) \text{ left } k \text{ units} \end{cases}$$

3. Vertical Expansion:

$$y = af(x) \begin{cases} a > 1 & \text{Vertically expand(stretch) the graph of } y = f(x) \text{ by multiplying each } y \text{ value by } a \\ 0 < a < 1 & \text{Vertically contract(compress) the graph of } y = f(x) \text{ by multiplying each } y \text{ value by } a \end{cases}$$

4. Horizontal Expansion:

$$y = f(bx) \begin{cases} b > 1 & \text{Horizontally contract(compress) the graph of } y = f(x) \text{ by multiplying each } x \text{ value by } \frac{1}{b} \\ 0 < b < 1 & \text{Horizontally expand(stretch) the graph of } y = f(x) \text{ by multiplying each } x \text{ value by } \frac{1}{b} \end{cases}$$

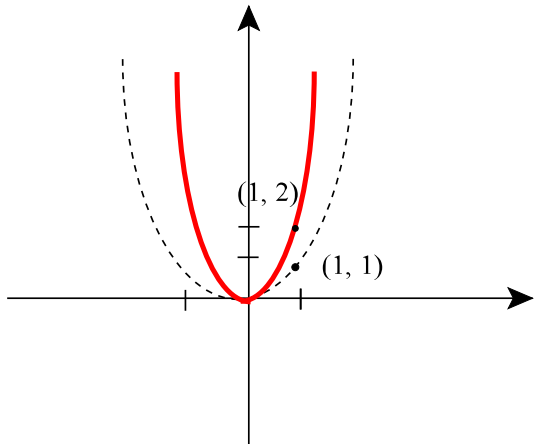
5. Reflection:

$$y = -f(x) \quad \text{Reflect the graph of } y = f(x) \text{ across the } x\text{-axis}$$

$$y = f(-x) \quad \text{Reflect the graph of } y = f(x) \text{ across the } y\text{-axis}$$

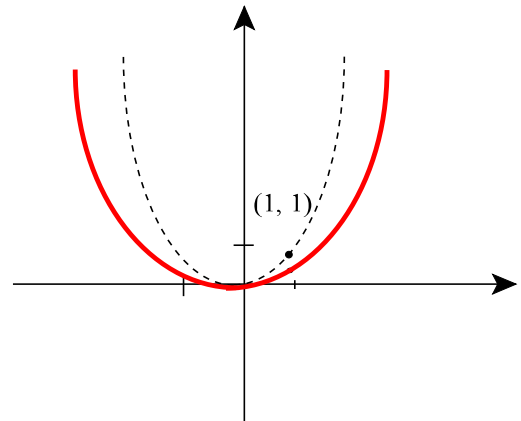
Example 1: To graph $y = 2x^2$, double the units along the y -axis.

(Expand $y = x^2$ vertically)



Example 2: To graph $y = \frac{1}{2}x^2$, half the units along the y -axis.

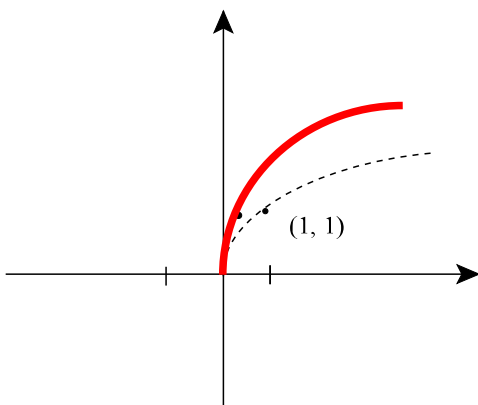
(Contract $y = x^2$ vertically)



Example 3: To graph $y = \sqrt{2x}$, half the units along the x -axis.

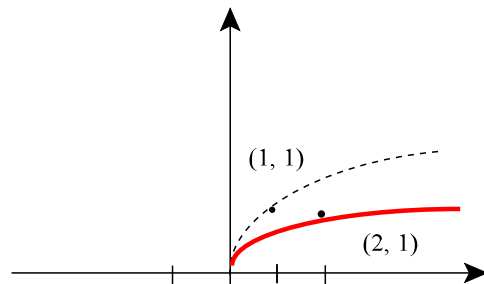
(Compress $y = \sqrt{x}$ Horizontally)

$(1, \frac{1}{2})$



Example 4: To graph $y = \sqrt{\frac{1}{2}x}$, double the units along the x -axis.

(Expand $y = \sqrt{x}$ Horizontally)



To graph $y = -\sqrt{x}$, reflect the graph of $y = \sqrt{x}$ in the x -axis. To graph $y = \sqrt{-x}$, reflect the graph of $y = \sqrt{x}$ in the y -axis.

