COLLEGE ALGEBRA, Math 1043

Abedi, Fall 2004 Name.....*KEY*.....

1. Find the he number of solutions of the following systems: $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$ *none*, $\left[\begin{array}{cccc} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right]$ *many*

2. Solve the system $\begin{cases} x + y = 2 \\ 2x - 3y = 9 \end{cases}$.

$\{(3, -1)\}$

3. Solve the system of linear equations $\begin{cases} x - 2y + z = 6 \\ x + y - 2z = -6 \\ 2x + y - z = -2 \end{cases}$.

$\{(1, -1, 3)\}$

$$\left[\begin{array}{cccc} 1 & -2 & 1 & 6 \\ 1 & 1 & -2 & -6 \\ 2 & 1 & -1 & -2 \end{array}\right] \xrightarrow{-R_1+R_2} \left[\begin{array}{cccc} 1 & -2 & 1 & 6 \\ 0 & -3 & 3 & 12 \\ 2 & 1 & -1 & -2 \end{array}\right] \xrightarrow{-2R_1+R_3}$$

$$\left[\begin{array}{cccc} 1 & -2 & 1 & 6 \\ 0 & -3 & 3 & 12 \\ 0 & 5 & -3 & -14 \end{array}\right] \xrightarrow{5R_2+3R_3} \left[\begin{array}{cccc} 1 & -2 & 1 & 6 \\ 0 & -3 & 3 & 12 \\ 0 & 0 & 6 & 18 \end{array}\right]$$

$$\begin{cases} x - 2y + z = 6 \\ -3y + 3z = 12 \\ 6z = 18 \end{cases}$$

$\boxed{z = 3}$, $-3y + 9 = 12$, $\boxed{y = -1}$

$x - (-2) + z = 6$, $\boxed{x = 1}$

4. Solve the following system of linear equations, if possible. If there are many solutions, write the general solution set.

i) $\begin{cases} x - 2y + z = 6 \\ x + y - 2z = -6 \\ 2x + y - z = -2 \end{cases}$.

$\{(1, -1, 3)\}$

$$\left[\begin{array}{cccc} 1 & -2 & 1 & 6 \\ 1 & 1 & -2 & -6 \\ 2 & 1 & -1 & -2 \end{array}\right] \xrightarrow{-R_1+R_2} \left[\begin{array}{cccc} 1 & -2 & 1 & 6 \\ 0 & -3 & 3 & 12 \\ 2 & 1 & -1 & -2 \end{array}\right] \xrightarrow{-2R_1+R_3}$$

$$\begin{bmatrix} 1 & -2 & 1 & 6 \\ 0 & -3 & 3 & 12 \\ 0 & 5 & -3 & -14 \end{bmatrix} \xrightarrow{5R_2+3R_3} \begin{bmatrix} 1 & -2 & 1 & 6 \\ 0 & -3 & 3 & 12 \\ 0 & 0 & 6 & 18 \end{bmatrix}$$

$$\xrightarrow{5R_2+3R_3} \begin{bmatrix} 1 & -2 & 1 & 6 \\ 0 & -3 & 3 & 12 \\ 0 & 0 & 6 & 18 \end{bmatrix} \qquad \boxed{z = 3}, \; -3y + 9 = 12, \boxed{y = -1}$$

$$x - (-2) + z = 6, \boxed{x = 1}$$

ii) $\begin{cases} 3x + y + 3z = 1 \\ x + 2y - z = -2 \\ 2x - y + 4z = 4 \end{cases}$.

None

iii) $\begin{cases} x - y + z = 0 \\ x + y + z = 1 \\ 2x + 2z = 1 \end{cases}$.

Many $\{(\frac{1}{2} - z, \frac{1}{2}, z) / z \in R\}$

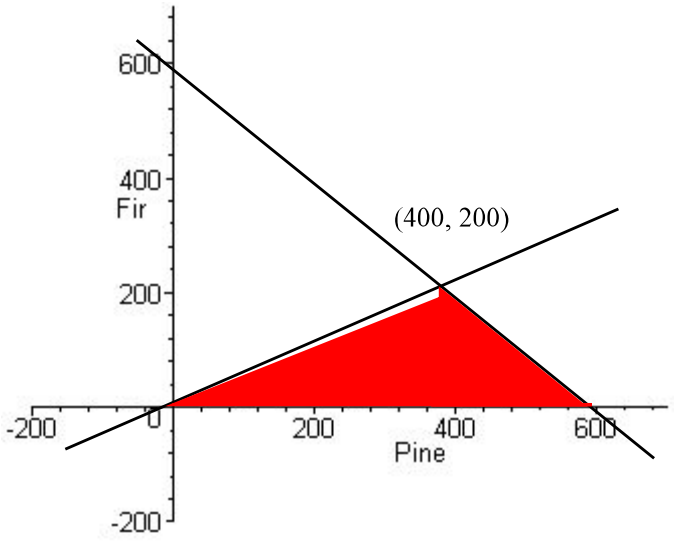
5. A tree farmer is getting ready to plant a crop of Christmas trees. She has room for no more than 600 trees total and wishes to plant both pine and fir trees. She knows from past experience that she must plant at least twice as many pines as fir trees to(e.g, for each Fir tree there should be two or more Pine trees) meet demand. The pine trees bring a profit of $10 each and fir trees bring a profit of $14 each. She wishes to know how many of each species of tree to plant to maximize her profit. Let x and y represent the number of Pine and Fir trees respectively

a) State your objective function, P, in terms of the variables

$P = 10x + 14y$

b) List the constraints that must be used, as inequalities, in order to solve the problem.

$$\begin{cases} x \geq 0, y \geq 0 \\ x + y \leq 600 \\ x \geq 2y \end{cases}$$



c) Graph the feasible region for this problem. Clearly label each of the vertices and axes.

d) Find the maximum profit subject to the constraints given.

$P = 10(400) + 14(200) = 6800$

$6800.00 @ (400, 200)