Math 1043. College Algebra,

Dr. Abedi, Fall 2004 *KEY*

1. Solve $2^{-x} = 16$, $x = -4$ and $2^{-x} = 14$ for x. Check your answerers.

$$-x = \log_2 14, x = -\frac{\log 14}{\log 2} = -3.8$$

2. Calculate $\log 10{,}000 = 4$, $\ln 300 = 5.738$, $\log_3 45 = \frac{\log 45}{\log 3} = 3.465$

3. Assume that $\log_b 3 = 0.613$ and $\log_b 5 = 0.898$ for some base b. Evaluate $\log_b 15$.

$$\log_b 15 = \log_b 5 + \log_b 3 = .898 + .613 = 1.511$$

4. Write $\log(x-1) + \log(x+1)$ as the logarithm of a single quantity. $\log(x-1)(x+1) = \log(x^2-1)$

5. Express the equation $5^3 = 125$ in logarithmic form. $\log_5 125 = 3$

6. Express the equation $\log_2 16 = 4$ in exponential form.. $2^4 = 16$

7. $1500 is deposited in a savings account with an annual interest rate of 7.5%. If the interest is compounded continuously, how long will it take for the value of the account to be $3000, $4500 ?

$$3000 = 1500e^{.075t}, 2 = e^{.075t}, \ln 2 = .075t, t = 9.24 years$$

$$4500 = 1500e^{.075t}, \frac{45300}{1500} = e^{.075t}, \ln\frac{45}{15} = .075t, t = 14.65 years$$

8. Solve $3^{x+2} = 93$ for x and write your answer in two decimal places.

$$x + 2 = \log_3 93, x = -2 + \log_3 93 = 2.13$$

9. Solve $\log(x-3) = 1$ for x and check your answer. $10^1 = x - 3, x = 10 + 3, x = 13$

10. The population of certain city was 15000 in 1995 and has been growing exponentially at the relative growth rate of 1.7 % per year. (hint: exponential growth model is given by $P(t) = 15000e^{.017t}$)

i) Estimate the population in the year 2001. $2001 - 1995 = 6, P(6) = 15000e^{(.017\times 6)} = 16611$

ii) At what year will the population reaches 40,000

$$40000 = 15000e^{.017t}, \frac{40000}{15000} = e^{.017t}, \ln\frac{40}{15} = .017t, t = 57.7 years$$

11. Solve the equation $\log(x-3) + \log(x+6) = 1$. (Don't forget to check your answer(s)).

$$\log(x-3)(x+6) = 1, \log(x^2+3x-18) = 1, x^2+3x-18 = 10^1, x^2+3x-28 = 0, x = 4$$

Not that x = – 7 does not check

12. Determine the hydrogen ion concentration of a sample of blood with pH = 7.41

$$pH = -\log[H^+], 7.41 = -\log[H^+], \log[H^+] = -7.41, [H^+] = 10^{-7.41} = 3.89 \times 10^{-8}$$

13. How long will it take $6000 to triple if it is invested in a savings account that pays 5.5% annual interest compounded continuously? Round to the nearest year.

$$18000 = 6000e^{.055t}, \ln 3 = .055t, t = \frac{\ln 3}{.055} = 19.97 \, years$$

14. Find the present value of $10,000 if interest is paid at a rate of 9% per year, compounded semiannually, for three years.

$$10000 = P\left(1 + \frac{.09}{2}\right)^{(2\times 3)}, 10000 = P(1.045)^6, P = \$7678.96$$

15. Find the amount invested at an annual interest rate of 9% if after 5 years of continuous compounding, the balance is $18,819.75.

$$18819.75 = Pe^{(.09\times 5)}, P = \$12000$$

16. The number of fish of a certain species is given by the formula $n(t) = 12e^{0.012t}$ where t is measure in years and $n(t)$ is measured in millions.

(a) What is the relative growth of the fish population? Express your answer as a percentage. *1.2%*

(b) What is the initial population of fish? *12*

(c) What is the population of fish in ten years? $P = 12e^{(.012\times 10)} = 13.53 million$

(d) How long will it take to 20 million fish? $20 = 12e^{(.012\times t)}, \frac{20}{12} = e^{.012t}, \ln\frac{20}{12} = .012t, t = 42.57 \, years$

(e) How long will it take to 40 million fish? $40 = 12e^{(.012t)}, \ln\frac{40}{12} = .012t, t = 100.33 \, years$

17. An earthquake has a energy of 1.78×10^{15} joules. Find its measure on the Richter scale in two decimal places.

$$M = \frac{2}{3}\log\left(\frac{1.78\times 10^{15}}{10^{4.4}}\right) = 7.23$$