College Algebra , Math 1043
Study Guide, Test 2

The following is a guide to assist you in preparing for the second t test. The sample problems are intended to better define the topic listed. These are **NOT** the only problems which could be asked on the topic. There probably will be study sessions for this second test. Also tutoring is available in Harris Hall Room 329 every day 8:00-4:00, except on Friday 8:00-12:00.

Quadratic Functions

1. $f(x) = ax^2 + bx + c, \quad a \neq 0$ Vertex = $\begin{cases} x = \dfrac{-b}{2a} \\ y = f(\dfrac{-b}{2a}) \end{cases}$

2. Complete the square to put in standard form.

$f(x) = a(x-h)^2 + k$

Example: Find the vertex of a parabola $f(x) = -2x^2 + 8x - 6$

x-coordinate of the vertex	h	
y-coordinate of the vertex	k	Vertex = $\begin{cases} x = \dfrac{-b}{2a} = \dfrac{-8}{2(-2)} = 2 \\ y = f\left(\dfrac{-b}{2a}\right) = f(2) = -2(4) + 8(2) - 6 = 2 \end{cases}$
Axis of symmetry	$x = h$	Graph this parabola and find the intercepts.

One-to-one functions

No two coordinate pairs in f have the same second coordinate and different first coordinates.

Graphs of one-to-one functions pass a **horizontal line test**.

Invertible Functions Must be one-to-one

1. Verify that $f(x)$ is one-to-one.
2. Set $y = f(x)$
3. Replace x with y.
4. Solve for y in terms of x and set y equal to $f^{-1}(x)$.
5. The domain of $f^{-1}(x)$ is the range of $f(x)$.
6. The range of $f^{-1}(x)$ is the domain of $f(x)$.
7. Graphs of $f(x)$ and $f^{-1}(x)$ are symmetric to $y = x$

Example: Graph the function $f(x) = 2x - 4$ and its inverse with axis of symmetry.

i) Set $y = 2x - 4$

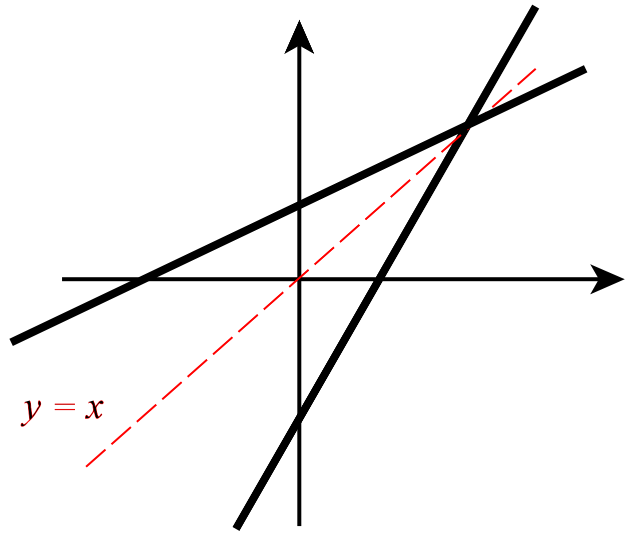
ii) Replace x with y.

$x = 2y - 4$

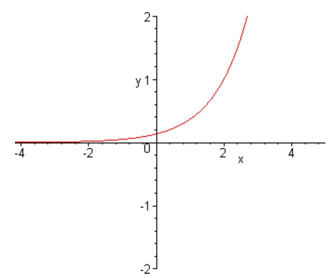
iii) Solve for y in terms of x and set y equal to $f^{-1}(x)$.

$2y = x - 4$, $y = \dfrac{x-4}{2}$, $f^{-1}(x) = \dfrac{x-4}{2}$

Note the graphs of $f(x)$ and $f^{-1}(x)$ are symmetric about the line $y = x$.



Exercise: Draw the inverse of the 1-1 function on the right.



Exercise: Which of the following functions are one-to-one?

$$g(x) = \sqrt{x} \quad , \quad h(x) = 3x - 1 \quad , \quad f(x) = x^2 - x - 2$$

Exercise:

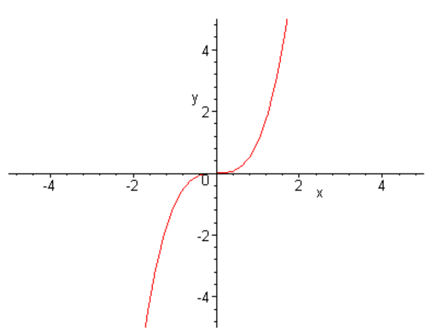
Find the inverse functions for the functions:

$$f(x) = \frac{x+2}{x-4} \quad , \quad h(x) = 3x - 1 \quad , \quad g(x) = \sqrt{x} \quad , \quad F(x) = x^2 \ , \ x \geq 0$$

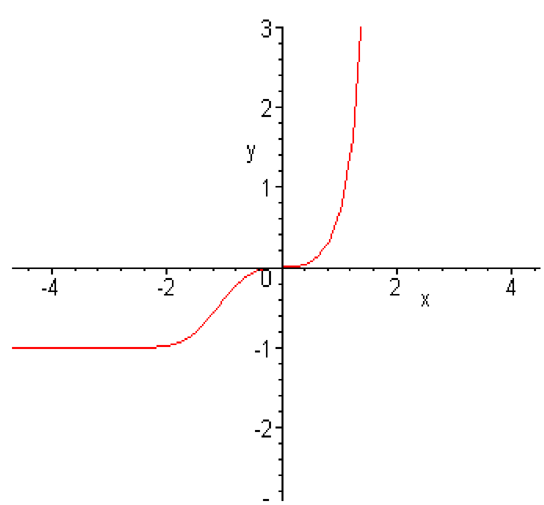
Examples: Determine whether the following are 1-1 functions.
(Horizontal Line Test)

Exercise:

Draw f^{-1}, the inverse of the 1-1 function f, on the same coordinate axes.



1-1



Not 1-1

Polynomial Functions

1. Know what a polynomial is and know the general form of the equation of a polynomial. Be able to determine the end behavior of a polynomial and be able to find: zeros, intercepts, local extrema, and know how to sketch the graph of a polynomial. The following is a typical example:

Example: Consider the polynomial $P(x) = x^3 + 3x^2 - 4x - 12$. Find: The leading coefficient of, the degree of the polynomial, the right-hand and the left-hand behavior of the graph of the polynomial, the real zeros (x-intercepts) of this polynomial, the y-intercept of the polynomial, and sketch the graph of this polynomial.

i) $n = 3$ (Degree)

ii) $a = 1$ (Lead Coefficient

iii) Rises to the right or increases without bound as $x \to \infty$,

falls to the left or decreases without bound $x \to -\infty$

iv) Zeros: $x = 2$, $x = -2$ $x = -3$ (x-intercepts)

v) Local Maximum: $(-2.53, 1.13)$, Local Minimum: $(.53, -13.13)$

vi) Exercise: Graph the polynomial. Find the y-intercept.

Exercises:

1. Which one of the following is a polynomial?

a) $P(x) = 5x^6 - 2x^5 + 12x^3 + 2x - 7$

b) $Q(x) = 3x^{\frac{4}{7}} - 2x^{\frac{1}{2}} + 2x - \frac{1}{7}$

c) $S(x) = \sqrt{12x^3 + 2x - 7}$

d) $Z(x) = \dfrac{6x^3 - 4x^2 + 3}{x - 1}$

For problems 2-4 let $f(x) = -20x^4 + 5x^3 - 3x + 8$.

2. Find the y-intercept of f________________

3. Find the degree of f ________________

4. Find the value of $f(-10)$________________

5. Sketch the graph of $y = x(x+2)^2(x-3)$.

Find and label the x- and y- intercepts. ________________

Find the degree of this function________________

1. Know the meaning of the **Location Theorem**, and be able to use that theorem to determine whether a polynomial has a zero in a given interval **or** a polynomial equation has a root in a given interval.

Exercises:

i. Show that the following polynomials have at least one **zero** in the given intervals:

$$P(x) = 2x^2 + x - 3\ ,\ [-2,-1] \qquad , \qquad P(x) = 2x^3 - 3x^2 + 2x - 3\ ,\ [1,2].$$

ii. Show that the following polynomials equations have at least one **root** in the given intervals:

$$3x^3 - 11x^2 - 14x = 0\ ,\ [4,5] \qquad , \qquad 5x^3 - 9x^2 - 4x + 9 = 0\ ,\ [-1,2]$$

iii. Let $P(x)$ be a polynomial whose some of its values at various points are given in the table below. What can you say about the zeros of this polynomial in the given range?

x	-8	-2	0	2	4	9
$P(x)$	-3	1	5	3	-2	25

Rational Functions

1. Know what a rational function is.

2. Be able to find the domain of a rational function:

3. Know what an asymptote of a rational function is.

Be able to find the vertical and the horizontal asymptotes of a rational function.

Exercise: Find the vertical and the horizontal asymptotes of rational functions:

$$f(x) = \frac{x^2 - x - 2}{x^2 - 4x + 3} \qquad f(x) = \frac{x^2 + 2x - 3}{x^3 - 4x} \qquad f(x) = \frac{2x - 1}{x + 3} \qquad f(x) = \frac{3x + 2}{x^2 - 4} \qquad f(x) = \frac{x + 2}{x - 4}$$

4. Be able to find the x-intercepts and the y-intercept of a rational function.

Exercise: Find the x-intercepts and the y-intercept of the rational functions in the above exercise:.

5. Be able to sketch or recognize graphs of rational functions.

Exercise: Sketch the graphs of functions in the above exercise:.